Chapter 2 Problems

1. A thin sheet of iron is in contact with a carburizing gas on one side and a decarburizing gas on the other at temperature of 1000°C.
   (a) Sketch the resultant carbon concentration profile when a steady state has been reached assuming the surface concentration are maintained at 0.15 and 1.4 wt% C.
   (b) If $D_c$ increases from $2.5 \times 10^{-11}$ m$^2$/s at 0.15% C to $7.7 \times 10^{-11}$ m$^2$/s at 1.4% C what will be the quantitative relationship between the concentration gradients at the surfaces?
   (c) Estimate an approximate value for the flux of carbon through the sheet if the thickness is 2 mm (0.8 wt% C = 60 kg/m$^3$ at 1000°C).

2. It was stated in Section 2.2.1 that $D = \Gamma \alpha^2 \pi/6$ applies to any diffusing species in any cubic lattice. Show that this is true for vacancy diffusion in a pure fcc metal. (Hint: consider two adjacent \{111\} planes and determine what fraction of all possible jumps result in the transfer of a vacancy between the two planes. Is the same result obtained by considering adjacent \{100\} planes?)

3. A small quantity of radioactive gold was deposited on the end of a gold cylinder. After holding for 24 h at a high temperature the specimen was sectioned and the radioactivity of each slice was as follows:

<table>
<thead>
<tr>
<th>Distance from end of bar to center of slice (µm)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity</td>
<td>83.8</td>
<td>66.4</td>
<td>42.0</td>
<td>23.6</td>
<td>8.74</td>
</tr>
</tbody>
</table>

Use the data to determine $D$.

4. Prove by differentiation that Equation 2.20 \[ C = C_0 \sin(\pi x/l) \exp(-t/\tau) \] is a solution of Fick’s second law.

5. Fourier analysis is a powerful tool for the solution of diffusion problems when the initial concentration profile is not sinusoidal. Consider for example the diffusion of hydrogen from an initially uniform sheet of iron. If the concentration outside the sheet is maintained at zero the resultant concentration is initially a top-hat function. Fourier analysis of this function shows that it can be considered as an infinite series of sine terms:

\[
C(x) = \frac{4C_0}{\pi} \sum_{i=0}^{\infty} \frac{1}{2i+1} \sin \left( \frac{2i+2}{l} \pi x \right)
\]

where

- $l$ is the thickness of the sheet,
- $C_0$ is the initial concentration.

a. Plot the first two terms of this series. If during diffusion the surface concentration is maintained close to zero each Fourier component can be considered to decrease exponentially with time with a time constant $\tau_i = l^2 / (2i+1)^2 \pi^2 D$. The solution to the diffusion equation therefore becomes

\[
C(x,t) = \frac{4C_0}{\pi} \sum_{i=0}^{\infty} \frac{1}{2i+1} \sin \left( \frac{2i+2}{l} \pi x \right) \exp \left( -\frac{t}{\tau_i} \right)
\]
b. Derive an equation for the time at which the amplitude of the second term is less than 5% of the first term.

c. Approximately how long will it take to remove 95% of all the hydrogen from an initially uniform plate of α-iron at 20°C if (1) the plate is 10 mm thick and if (2) it is 100 mm thick, assuming the surface concentration is maintained constant at zero? (Use data in Table 2.1.)

<table>
<thead>
<tr>
<th>Solute</th>
<th>(D_0/\text{mm}^2 \text{s}^{-1})</th>
<th>(Q/\text{kJ mol}^{-1})</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.0</td>
<td>84.1</td>
<td>2</td>
</tr>
<tr>
<td>N</td>
<td>0.3</td>
<td>76.1</td>
<td>3</td>
</tr>
<tr>
<td>H</td>
<td>0.1</td>
<td>13.4</td>
<td>4</td>
</tr>
</tbody>
</table>

6. Figure 2.31 shows the molar free energy-composition diagram for the A-B system at temperature \(T_1\). Imagine that a block of \(\alpha\) with composition (1) is welded to a block of \(\beta\) phase with composition (2). By considering the chemical potentials of the A and B atoms in both the \(\alpha\) and \(\beta\) phases predict which way the atoms will move during a diffusion anneal at \(T_1\). Show that this leads to a reduction of the molar free energy of the couple. Indicate the compositions of the two phases when equilibrium is reached.

7. A diffusion couple including inert wires was made by plating pure copper on to a block of \(\alpha\)-brass with a composition Cu-30 wt% Zn, Fig. 2.20. After 56 days at 785°C the marker velocity was determined as \(2.6 \times 10^{-8} \text{ mm/s}\). Microanalysis showed that the composition at the markers was \(X_{\text{Zn}} = 0.22\), \(X_{\text{Cu}} = 0.78\), and that \(\partial X_{\text{Zn}}/\partial x\) was 0.089 mm\(^{-1}\). From an analysis of the complete penetration curve \(\overline{D}_\alpha\) at the markers was calculated as \(4.5 \times 10^{-13} \text{ m}^2/\text{s}\). Use this data to calculate \(D_{\text{Zn}}^\alpha\) and \(D_{\text{Cu}}^\alpha\) in brass at 22 atomic % Zn. How would you expect \(D_{\text{Zn}}^\alpha\), \(D_{\text{Cu}}^\alpha\) and \(\overline{D}_\alpha\) to vary as a function of composition?
8. Draw possible free energy-composition curves for the system in Figure 2.29 at \( T_1 \). Derive from this a \( \mu_B-X_B \) and an \( a_B-X_B \) diagram (similar to Figure 1.28). Mark the points corresponding to \( p \) and \( q \) in Figure 2.29c. Sketch diagrams similar to Figure 2.29c to show \( a_A, \mu_A, \) and \( \mu_B \) across the diffusion couple. What will be the final composition profile when the couple reaches equilibrium if the overall composition lies (1) between \( a \) and \( b \), (2) below \( a \)?

9. Figure 2.32 is a hypothetical phase diagram for the A-B system. At a temperature \( T_1 \) B is practically insoluble in A, whereas B can dissolve 10 at% A. A diffusion couple made by welding together pure A and pure B is annealed at \( T_1 \). Show by a series of sketches how the concentration profiles and \( \alpha/\beta \) interface position will vary with time. If the overall composition of the couple is 50 at% B what will be the maximum displacement of the \( \alpha/\beta \) interface? (Assume \( \alpha \) and \( \beta \) have equal molar volumes.)